Federalist 10 and the Chaos Theorem, Part I

by JAMES R. ROGERS

Kenneth Arrow famously demonstrated that, under a set of conditions, no system of majority voting can guarantee the outcome of voting by entirely rational voters will itself be rational. Or to put it in a pithier form, no voting procedure can guarantee that individually rational voters will result in socially rational outcomes. I chat about what this means below. Modern political scientists and economists worry about this result a lot, and many think it represents a big problem for democracy. Interestingly, however, James Madison draws on a similar intuition in Federalist 10 to argue the same dynamic is a good thing for democracy. And modern constitutional jurisprudence, borrowing from Madison, also suggests it’s a good thing. In this post I review Arrow’s argument and some of its extensions. In my next post, I plan to chat about Madison’s and the Supreme Court’s treatment of the same implication.

Some brush clearing first, however. As an initial matter, there is the oft-misunderstood word, “rationality.” Being “rational” is a thick concept in everyday parlance. In rational choice theory, however, rationality is a very thin concept. In everyday language, a person is irrational if a person’s understanding does not fundamentally comport with agreed upon reality. A person who thinks himself or herself to be Napoleon (and is in fact not Napoleon), is termed “irrational.”

Not for the rational choice theorist, however. Rationality in rational choice theory pretty much means two specific things: First, choices over a set of outcomes are “complete.” This means a person (or institution) can rank any two outcomes, A and B, as “A is preferred to B,” “B is preferred to A,” or “A is indifferent to B.” The second component of rationality is that preferences are transitive. If a person or institution prefers A to B, and B to C, then that person prefers A to C.
That’s all it means to be rational in rational choice theory. So a person who’s not Napoleon can sincerely believe he’s Napoleon, but as long as that person’s preferences are complete and transitive, then rational choice theory defines that person as rational. Or, at least, defines that person’s choices as rational. The failure to understand the very thin notion of rationality used in “rational choice theory” provides genesis to a good number of off-target objections to rational choice theory. Most crazy people are entirely rational the way rational choice theory defines rational.

Back to Arrow.

The key implication of Arrow’s impossibility theorem is voting by a group of entirely rational voters (as defined above) does not guarantee the voting outcome will be rational (as defined above). Specifically, when three or more voters face three or more choices, the majority-vote “social choice” of the group cannot be guaranteed to be rational. This despite each individually being entirely rational.

The rub is transitivity, one of the two elements defining rationality. The problem can be easily illustrated. Take three individually rational voters. They’re friends, and are trying fairly to decide the best pie to make to take on their picnic. Voter 1 prefers apple pie to blueberry, and blueberry to cherry. Because voter 1 is rational, voter 1’s preferences are transitive. So voter 1 prefers apple pie to cherry pie. Voter 2 prefers blueberry pie to cherry, and cherry to apple, etc. Voter 3 prefers cherry pie to apple and apple to blueberry, etc.

The social choice of these three completely rational friends through a majority vote, however, will be irrational, that is, intransitive. (We’re assuming sincere, pairwise voting, yadda, yadda, yadda.)

In a majority vote between choosing apple pie or blueberry pie, apple pie wins by a vote of two to one. In a majority vote between blueberry and cherry, blueberry wins. If the friends’ social choice were transitive, then the group would prefer apple pie to cherry pie. But in a vote between apple and cherry, cherry wins the majority. The social preference of our small society of three friends is intransitive, that is, irrational. And this despite the individual preferences of the three friends being entirely rational.

This is called a voting cycle. Every type of pie can result as a majority selection. The type of pie actually selected will result from who sets the agenda, or when they get tired of voting. Both are arbitrary decisions with respect to the social preference over pie.

Economists and political scientists began to worry about the implications of this result for democracy soon after Arrow began publishing on the theorem. In 1967, however, Gordon Tullock published an article in the Quarterly Journal of Economics titled, “The General Irrelevance of the General Possibility Theorem.” In the article Tullock rejected the idea that voting cycles posed a problem for democracy. Tullock argued we need not worry much about voting cycles because, when they exist, cycles generally will be confined to small areas in the policy space. Cycles exist, he argued, but they’re largely irrelevant.

Tullock argued by illustration, however, rather than systematically. Nine years later, prompted by Tullock’s QJE article, Richard McKelvey published an article in the Journal of Economic Theory, “Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control.” In this article McKelvey demonstrated, when cycles exist, sincere, pairwise majority voting can lead to any outcome in the policy space, including even points outside of the voters’ Pareto set. Contra Tullock, voting cycles were not intrinsically confined to local areas in the policy space.

The result is called the “chaos theorem” because majority voting can lead to any policy outcome in the policy space. This name is a somewhat misleading as well. Readers, including not a few academics, take the
result to suggest that policy outcomes under majority rule will be randomly hither and yon, that they will be unpredictable “chaos.” Not so. McKelvey’s result only suggests that an agenda setter can lead a group of voters anywhere in the policy space. Agenda setters would, of course, want to lead the voters to the agenda setters’ own preferred policy outcome. To be sure, however, agenda setters can and do change. In an ordinary legislative body, for example, different members may propose different policies, and may propose amendments to policies being considered. McKelvey’s theorem means any proposal can be majority-supported by some series of majority votes. Outcomes are not chaotic in the way the word is often used.

The reason Arrow’s impossibility theorem and McKelvey’s chaos theorem received the attention they did (and do) was because it sounds bad to say that democracy cannot guarantee rational policy outcomes. Kenneth A. Shepsle, for example, first received wide attention pointing out how institutions, such as legislative committees, can induce stability over otherwise chaotic majority outcomes. William Riker and Barry Weingast suggest the need for heightened judicial review to limit deleterious implications of majoritarian chaos in legislatures. Economists and political scientists routinely treat these results as undermining the case for democratic decision-making.

Funny thing, though, James Madison and US Supreme Court jurisprudence seem to treat majoritarian indeterminacy as a good thing rather than a bad thing. I’ll consider their arguments in light of the chaos theorem in my next post. And muse over who might have the better argument, if any one.

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[...] insight that majoritarian decision making need not be rational. As I’ve discussed before (here and here, among other places) by arguing that majoritarian outcomes are “irrational” social [...]