Local Government Valuation*

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Abstract

We construct a novel data set on the fiscal position of municipalities in the United States and document a secular decline in their financial health. Our data combines financial data from the Annual Comprehensive Financial Reports (ACFRs) of municipalities along with Census data of their revenue and expenditure cash flows. We find that 61% of municipalities operate with a negative net position—akin to a negative book equity position in the corporate context. We find that most of the decline originates from the accumulation of legacy obligations, i.e., pensions and other post-employment benefits (OPEBs); this is recognized by municipal bond markets through higher credit spreads. Since accounting values are backward looking, we turn to the market valuation of local governments’ equity by estimating an SDF that matches the valuation of a wide range of assets in the economy to price untraded future tax and expenditure claims. We find that the market values of equity are highly correlated with the book values. The negative equity position—in terms of book and market values—for some local governments suggests the presence of implicit insurance by state and federal governments. Our results suggest that as many as half the municipalities with negative market value of equity would need 10 years of higher government intervention.

Keywords: Municipal finance, financial distress, public finance, fiscal sustainability, sovereign default, asset pricing.

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1 Introduction

The national accounts of the United States emphasize flow over stock variables. As such they mostly ignore the assets and liabilities of sub-national government entities even though these governments maintain a balance sheet similar to corporations.\(^1\) As a result, very little is known about the capital structure and the solvency of local governments. This is despite the economic importance of local governments in the U.S. They account for $1.6 trillion — 8.1% of U.S. GDP — in public expenditures and 10% of total non-farm employment.\(^2\) In addition, they perform a number of important functions in public works, public safety, and other local public amenities.

This paper provides a systematic study of the balance sheet of city governments in the United States. The information about assets and liabilities offer an understanding of the capital structure and crucially the equity position of governmental entities. The equity position, measured as the difference between the assets of an entity against its liabilities, represents the net worth of the local government, analogous to the equity position of corporations, thereby providing an assessment of its financial health. Specifically, we are able to ask: What is the financial health of U.S. local governments, and how has their health evolved over time? The relevance of these questions is apparent against the backdrop of relief measures in the wake of the COVID-19 pandemic. The CARES Act, FFCRA, RRA, and ARPA disbursed $415 billion to local governments despite incomplete information about the financial conditions and the fiscal impact of the pandemic (Clemens, Hoxie, and Veuger, 2022). The mere risk of potential financial fragility and the associated employment and essential services loss led to the largest transfer of resources to local governments in U.S. history.

We proceed through two major steps. First, we measure the financial health of local governments from existing data. This is a non-trivial task because annual financial re-

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\(^1\)Governmental entities are legally required to file an annual report. However, the annual reports are in an unstructured data type and come in heterogeneous formats; thus, making any systematic collection and comparison difficult.

\(^2\)The Annual Survey of State and Local Government Finances (ASSLGF) serves as the basis for the national accounts. The ASSLGF does not collect information on governmental assets, pension obligation, OPEB, and other obligations. The ASSLGF has some debt measures. However, we found only a weak correlation with the debt amounts that were disclosed in the ACFRs. The Financial Accounts of the United States, L.107, collect aggregate statistics for state and local governments which masks the cross-sectional heterogeneity. Furthermore, the information is insufficient to re-construct the balance sheet of governmental entities.
ports of local governments in the United States exist in a decentralized and unstructured way, with little to no data harmonization. To systematically capture the capital structure of local governments, we draw on a novel data set of the fiscal position of municipalities in the United States. The data set consists of balance sheet information from the annual comprehensive financial reports (“ACFRs”), which includes detailed measures on pension obligations, other post-employment benefits (“OPEB”) obligations, outstanding debt, as well as, capital assets. In addition, we obtain a long time series on revenue and expenditures of city governments from the Census Bureau. Overall, this data set provides a comprehensive picture on the financials of these governments for both flow and stock variables. Specifically, in this paper, we focus on the equity position of city governments. We gather data on 1,800 city governments.

We document three important facts: First, as Figure 1a shows, 61% of municipalities operate with a negative unrestricted net position—akin to a negative book equity position in the corporate context—and the share has increased over the past ten years. Second, this fiscal position is strongly associated with the accumulation of legacy obligations, such as pensions and other post-employment benefits. Third, markets recognize this deterioration in the fiscal position by demanding higher yields on municipal debt securities that

![Graph](image-url)
have worse fiscal positions, as can be seen in Figure 1b. However, the magnitude of the difference in spreads, while statistically significant, is economically small.

We take the surprising observation that a large share of city governments operate with a negative book equity position as motivation for the second major step in this paper: to study the fiscal position through market valuations. Thus far, we used accounting values from the ACFRs to value assets and liabilities. Accounting values come with the drawback that they are predominantly backward looking. As such, accounting values have only limited information content on how the financial position of the city government reflects its future economic prospects. Thus, we study the market value of the equity position of local governments. Market values are forward looking and thus alleviate the biggest shortcoming of book values. Assessing the market value of equity requires a disciplined way of valuing revenue and expenditure claims of local governments as their market value is not directly observed. We price these untraded claims by deriving their valuation from an estimated stochastic discount factor.

To provide a better sense of how we measure the financial health of local governments, consider the following illustration. Local governments in the United States have rich cross-sectional variation in their sources of revenues and expenditures. For example, some municipalities receive a relatively large fraction of revenues from property taxes, while others do not, instead receiving revenues primarily from sales taxes and other charges. This is a meaningful distinction. Because property taxes are tied to property valuations that are re-assessed after intervals as different as 1 to 8 years, the property tax base is relatively insulated from short term business cycle fluctuations. Contrast this to sales taxes that are mechanically tied to short term GDP growth fluctuations. In fact, if we regress growth in own source revenue for municipalities on detrended growth in GDP, we see that municipalities with higher share of revenues from property taxes are relatively less exposed to business cycle fluctuations. This paper will take a disciplined approach to this idea, in the sense of measuring the risk exposure of different revenue and expenditure streams of a large panel of local governments across the United States.

Our estimated stochastic discount factor can fit a broad set of asset prices in the economy well, including a broad index of the municipal bond market. Thus, we are pricing the untraded claims consistent with observable prices for other assets in the economy. Using a long time series for the revenue and expenditure claims of a rich panel of 388
local governments, we estimate the cross-sectional exposure to systematic risk. We use the estimated valuation ratios for government expenditure and revenues to calculate the market value of equity for local governments. More precisely, we calculate the present value of revenues plus cash holdings on the asset side, and subtract out the present value of expenditures, the value of pension obligations and OPEBs, and the present value of debt on the liabilities side.

Fundamentally, one contribution of our paper is in capturing the differential exposure of local governments in the United States to aggregate risk through a stochastic discount factor. Apart from the absolute valuation similar to the valuation of government debt at the federal level (Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019), we additionally obtain relative valuations in the cross-section of municipalities. In the presence of a rich cross-section of local governments, this allows us to obtain relative valuations of the government purged of aggregate risk. The cross-sectional differences we observe are large and meaningful, even while being closely related to book valuations. This has important implications. The cross-sectional differences remove the effect of any mispricing at the aggregate level, since that would be true for all local governments. The relative valuations are particularly insightful since they remove the potential of other common omitted factors to affect the valuation. This includes e.g. the impact of inflation (Hilscher, Raviv, and Reis, 2021) or common convenience yield on sub-national debt analogously to the convenience yield observed for federal debt (Krishnamurthy and Vissing-Jorgensen, 2012; Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019), or common rational bubbles (Samuelson, 1954; Diamond, 1965; Brunnermeier, Merkel, and Sannikov, 2020).

An important methodological contribution of our paper is measuring the exposure of hundreds of local governments in the United States to aggregate risk. We price a large cross-section of claims by embedding the exposure in an exponentially affine asset pricing model. This is a non-trivial task and we contribute to the literature by showing how the log-linear pricing formulas, a key tractability feature of exponentially-affine models, extend to price projections of untraded claims on aggregate state variables. Subject to regularity conditions on the maximal Sharpe ratio implied by the estimated SDF and the projection coefficients, these formulas yield well defined prices. The projection also provides an orthogonal decomposition of local government revenues and expenditures due to aggregate and idiosyncratic variation.
Our results suggest that a non-trivial number of municipalities operate with *negative market value* of equity. On the face of it, this would suggest that a number of municipalities across the U.S. are insolvent. Our findings resonate with a growing literature that has questioned the sustainability of sovereign debt in the United States (Rubin, Orszag, and Sinai, 2004; Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019). Federal debt as a percent of gross domestic product has approximately doubled between the onset of the great financial crisis and 2021. However, important differences exist in our setting: local governments operate under a different set of constraints. Local governments are subject to budgetary constraints that limits their tax authority and ability to finance operating expenditures with additional debt issuance (Alt and Lowry, 1994; Bohn and Inman, 1996; Krane, Rigos, and Hill, 2001; Poterba, 1994, 1995; Reschovsky, 2019). These constraints limit the potential corrective actions that local governments can undertake to improve their financial position.

Despite having negative market values of equity and institutional constraints on taking corrective action, corresponding municipal yields are low and stable in the data. In fact, credit spread differences between positive equity value municipalities and negative equity value municipalities are small. To resolve this conundrum, we argue that a simple accounting of *visible* cash flows ignores the important role of implicit insurance provided by federal and state governments. Our approach assumes that market participants incorporate this implicit insurance when valuing the municipalities’ debt position, i.e., there is no mis-pricing in the municipal bond market. The low and stable yields in the municipal market, therefore, is a consequence of market participants taking into account implicit state and federal insurance. We conceptualize transfers by state and federal government as an option on the underlying position of the municipality (Merton, 1977). Our exercise in determining the implicit insurance provided by higher forms of governments provides an estimate of the size of intervention that would be required to bail-out insolvent municipalities. We model the intervention threshold in terms of a fraction of the yearly deficit of the city government. That is, an intervention is triggered when the deficit reaches an unsustainable level. We find that multiple years of intervention are required to rationalize the negative market value of equity in the presence of limited spread discrimination of the financial position in the municipal bond market. That is, higher governments have to cover the shortfall in the municipality’s revenues over multiple years. This result is vali-
dated by the prolonged intervention period for the cities for which we observed chapter 9 bankruptcy proceedings in the last two decades.\textsuperscript{3}

We rule out other possible mechanisms. First, it may be asked if increases in tax revenues could rationalize our findings. After all, local governments have taxation power. However, we show that the magnitude of tax increase necessary to cover the shortfall for the average municipality with negative equity value is greater than 20%. We consider such an increase highly improbable, even without considering the endogenous response of residents to out-migrate due to tax hikes, which would ultimately reduce the tax base (Giesecke and Mateen, 2022). Second, we have found no systematic pattern or precedent of sale of capital assets by the local government to raise revenues to redress their liabilities problem.\textsuperscript{4} In fact, capital assets of local governments are excluded in bankruptcy proceedings under Chapter 9. The provision to liquidate capital assets and distribute the proceeds to creditors is considered a violation of the Tenth Amendment to the Constitution (United States Courts, 2023).

Interventions by the federal or state government have varying degrees of visibility. Interventions that are conducted as part of a Chapter 9 bankruptcy proceeding tend to be salient. While fiscal distress is widespread, we observe only 37 Chapter 9 bankruptcies since 2000 (Duffy and Giesecke, 2023). Chapter 9 bankruptcy proceedings are often considered as a last resort. More common are preemptive interventions that remain little noticed beyond the governmental officials that are directly involved in the negotiations. For instance, Milwaukee, WI, received concession to obtain a larger share of the state’s sales tax revenues and was allowed to levy a supplementary local sales tax to pay for the unfunded pension liability that posed a serious risk of bankruptcy (AP News, 2023). Interventions by higher levels of governments, therefore, are more common than often perceived.

\textbf{Related Literature:} We build on the extensive literature on debt sustainability in macro-economics starting with the seminal work of Hansen and Sargent (1980), Hansen,  

\textsuperscript{3}For a detailed discussion of the Detroit Chapter 9 filing and the prolonged intervention and recovery process, see Giesecke (2022).

\textsuperscript{4}Perhaps the most prominent case was a proposal to sell the art collection at the Detroit Institute of Art in 2013/14. The museum refused to recognize that the art collection was part of the city’s assets and were prepared to litigate if necessary. Ultimately, the bailout “Grand Bargain” explicitly asked for the collection to be not sold, thereby preventing courts from ruling on the case (Forbes, 2013).
Roberds, and Sargent (1991), and Sargent (2012). This literature assumes mostly a constant discount rate and ignores the differential cyclical properties of claims. We adopt risk adjusted returns from the asset pricing literature (Alvarez and Jermann, 2005; Hansen and Scheinkman, 2009; Backus, Boyarchenko, and Chernov, 2018) that results from a stochastic discount factor that prices states of the world, while drawing on the extensive literature in public finance to value pension and other post employment benefits (Giesecke and Rauh, 2022; Brown and Pennacchi, 2016; Novy-Marx and Rauh, 2011a; Brown and Wilcox, 2009; Lucas and Zeldes, 2006).


Further, we contribute to the literature on local finances. Adelino, Cunha, and Ferreira (2017) studies the re-calibration of credit ratings and the associated change in fiscal capacity; Shoag, Tuttle, and Veuger (2019), Chava, Malakar, and Singh (2021a) study the fiscal implications of large bankruptcies on local communities; Cellini, Ferreira, and Rothstein (2010) study the effect of bond elections; Yi (2021) studies the credit supply shock in the municipal bond market due to financial regulation; Haughwout, Hyman, and Shachar (2021), Green and Loualiche (2020) and Clemens and Veuger (2021) study the fiscal implications of COVID-19 on state and local governments, Chava, Malakar, and Singh (2021b) studies the impact of business subsidies on municipal bond yields and Giesecke and Mateen (2022), Chernick, Reschovsky, and Newman (2021) study the effect of a decline in the local tax base. Carlson, Giammarino, and L Heinkel (2022) studies the optimal capital structure of a municipality trading off tax costs and bankruptcy costs. Boyer (2018) estimates the seniority of bond claims relative to pension claims and finds that bond claims are mostly junior to pension claims. Given this, we would expect municipal markets to demand a higher yield on city governments with worse financial health. We find this is not so, even for seemingly insolvent governments, and our implicit insurance exercise
provides a reconciliation of the evidence. Myers (2022) shows how municipal governments’ option to file for fiscal emergencies leads to perverse incentives into how they manage spending and borrowing, leading to excessive risk-taking. He incorporates the idea of a threshold where municipalities trigger bankruptcy based on their inability to service basic amenities. Our implicit insurance exercise captures the notion of these thresholds. We additionally incorporate the idea of the bailout being provided by higher levels of government, as against the residents of the municipality. As explained before, the magnitude of tax increase necessary to cover the shortfall in financial position is very large, and much larger than typical increases in taxes that local residents choose to vote in after default events. Indeed, Reschovsky (2019) documents how most local governments in the U.S. have limited institutional autonomy to increase taxes. Therefore, short of a systematic change in state constitutions across the U.S., possibly triggered by a wave of bankruptcies, a tax increase large enough to cover the fiscal shortfall we document in this paper is highly unlikely. Importantly, any tax increase needs to consider the endogenous response of local residents. As shown by Giesecke and Mateen (2022), under a quasi-experimental setting, local residents respond to tax rate increases by out-migrating, thereby further reducing the tax base and financial health of the local government.

A number of papers have explored the implications of institutional constraints on local government fiscal decisions (Alt and Lowry, 1994; Bohn and Inman, 1996; Krane, Rigos, and Hill, 2001; Poterba, 1994, 1995; Reschovsky, 2019). There is cross-sectional heterogeneity in these constraints. Some states offer more proactive policies for supporting local governments during times of crisis, which is reflected in municipal bond yields, as in Gao, Lee, and Murphy (2019). It has also been shown that local governments respond to negative fiscal shocks through reducing expenditure or increasing taxes depending on available margins and their independence to set policies, for example in “home rule” governments, as in Shoag, Tuttle, and Veuger (2019). Our paper implicitly captures the long term implications of this heterogeneity in local constraints by capturing its net effect on financial health.

The paper is organized as follows. Section 2 introduces our novel dataset along with pre-existing dataset that we use in our study. Section 3 documents important facts on the status quo and the trajectory of the financial conditions of municipalities across the United States and provides evidence of the cross-sectional pricing in the municipal bond
market. Section 4 discusses the structure of local governments’ balances sheets and its implications for the re-pricing. Section 5 introduces the pricing methodology, Section 6 conducts the market valuation of equity of U.S. municipalities and Section 7 concludes.
2 Data

We assemble a comprehensive dataset on the financial position of municipalities across the United States. We summarize the main components of our data here and provide further details in the Data Appendix.

2.1 Data and Sample Selection

Annual Survey of State and Local Government Finances The Annual Survey of State and Local Government Finances (ASSLGF) serves as a key input for the tabulation of the national accounts pertaining to the revenues and expenditures of all governmental units. The Census Bureau conducts a full census in years ending with "2" and "7"; and a survey of a subset in between. This includes the so called “certainty sample” which is surveyed in every year. The certainty sample constitutes the main sample for our analysis. We confirm that the information from ASSLGF are consistent with the national account from the Bureau of Economic Analysis (BEA). The validation for a selected set of items is shown in the Appendix in Figure DA.3, Figure DA.1, and Figure DA.2.

Moody’s Investors Service Data We augment the ASSLGF data with more detailed data on assets and liabilities from annual comprehensive financial reports (ACFRs). While municipalities are legally required to file ACFRs annually, the publication is irregular at best.\(^5\) Even after obtaining the reports, any comprehensive study is difficult because the reports are provided in an unstructured data format. We overcome the data scarcity by drawing on a large dataset on key financial indicators for a broad sample of local governments across the United States from Moody’s Investors Service. Moody’s prepares these financial indicators as part of its rating service by drawing directly on the ACFRs and subsequently harmonizing the records for comparison. The financial indicators serve as a primary input into the rating of municipal debt securities.\(^6\) We further augment the Moody’s data with manually collected data to obtain the best possible coverage for the

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\(^5\)For example, the state auditor of California launched a project in 2019 that—for the first time—collected the annual comprehensive financial reports from all its municipalities. This initiative has been started to create transparency and identify financial distress early—a recognition of increasing pension and other post employment obligations.

\(^6\)We validated the records for a random sample of municipalities and found them to be consistent with available ACFRs.
certainty sample.

**Municipal Bond Data**  We complement the data set on balance sheet conditions of local governments with municipal bond yields in the primary and secondary market. Primary market information are obtained from Mergent Municipal Bond Database which records issuer characteristics and a large set of bond characteristics. In addition, we obtain secondary market data from MSRB EMMA. MSRB EMMA is a trade repository which records every trade in municipal bond securities since 2005. MSRB EMMA includes the trade time, trade price and implied yield to maturity, as well as, information whether it was a broker to customer trade or broker to broker trade. We only include broker to customer trades in our analysis. One challenge is to establish the connection between the bond issuance and the bond issuer. While Mergent Bond Database records the issuer name, the match to the financial information of the issuing entity is not straightforward. We overcome part of the challenge by using Moody’s historical linkage table. This table is created as part of Moody’s ratings activity and documents in great detail whether a local government is the direct issuer or the financial obligor for an issuance. While this linking table covers many debt securities, it does so only for a subset. For the remainder we draw on the universe of debt security disclosures under the U.S. Securities and Exchange Commission (SEC) Rule 15c2-12 as provided by MSRB.

**Local Government Shape Files**  As local governments are not defined homogeneously across the United States, we create new shape files that represent the jurisdictional boundaries of local governments. More details on the historical reason for the heterogeneity across the United States, as well, as details on the construction are provided in Section DA.7.2 of the Data Appendix.
3 Financial Conditions Across U.S. Municipalities

How did the financial conditions of local governments evolve over the last decade? We document the secular development of financial health for a broad sample of local governments across the United States. As the accounting and reporting of financial indicators differs from corporations we provide some institutional background on the accounting methodologies and introduce the two main financial indicators that we focus on in Section 3.1. Section 3.2 presents the secular development in association with the market based measures of financial health for the national sample. Finally, Section 3.3 focuses on the sample that will be used in the market valuation exercise in the rest of the paper.

3.1 Institutional Background

Accounts and Accounting Local governments manage their finances typically based on governmental funds. The general fund covers most of the operational revenues and expenditures; other funds—such as, the capital project, debt service, internal service, and enterprise fund—often exist and take on a more specialized role. Every state except for Vermont imposes a statutory or constitutional balanced budget provision on the general fund (NCSL, 2010).

The accounting basis for these funds is fund accounting or modified accrual accounting. Fund accounting emphasizes cash-flows over the accrual of expenses and resembles the cash flow statement rather than the profit and loss statement or balance sheet in the corporate context. While the accounts are the primary basis for decision making, local governments are required to publish the statement of net position in their comprehensive annual financial report. The methodology for the statement of net positions is closer to conventional accrual accounting. The statement of net position represents assets and liabilities more comprehensively. However, the funds receive most of the attention in the administrative decision-making process. In principle, this hybrid accounting framework allows for large deficits on an accrual basis as long as it does not materially affect the (cash) balance in the general fund. Pension and other post employment benefit commitments are two examples for which the expenditures and the cash flow impact occurs with

\footnote{The balanced budget provision applies with varying degree of stringency as e.g. discussed in Bohn and Inman (1996) and Poterba (1995).}
a large time gap. Thus, the difference between operating expenses and the incrementally accrued liability can be large.

**Financial Indicators** We use two main financial indicators to describe the development of financial conditions of local governments. First, the unrestricted net position as a percentage of operating revenues, and second, the total debt as a percentage of the full value. These indicators are used as an input in ratings agencies’ methodology. The unrestricted net position is directly reported in the statement of net position of the ACFR and is an important input into the credit rating. The unrestricted net position consists of three major parts: (i) long-term debt that is not directly associated with capital assets, (ii) pension obligations, and (iii) other post employment benefits. The portion of long-term debt that is not directly associated with capital assets can be understood as debt that has been issued to fund operating expenses. The use of the unrestricted net position derives its justification under the premise that most of the capital assets are highly illiquid and thus cannot be used to serve the liabilities. It excludes the fraction of liabilities that are directly associated with the capital assets—revenue bonds to fund capital projects is one such example. Further, the unrestricted net position is calculated under an accounting framework that is closer to accrual accounting, that is, the expenditures are accounted for at the time of accrual, not at the time of the cash outflow. The total debt as percentage of the full value captures the indebtedness relative to the maximum amount of all taxable properties that could be drawn upon for taxation and is another prominent input into the rating of municipal securities.

We consider the aforementioned financial indicators to represent the overall financial position most accurately. For the purpose of the most direct comparison with the corporate balance sheet, we also consider the net position. The net position is the difference of all assets minus liabilities and is analogous to definition of book equity in the corpo-
rate context. In later sections, we will show how our independently constructed market valuations are positively related to the above financial indicators. We will also find our exercise captures additional variation not captured by book values.

3.2 Nationwide Sample

This section describes the state and the trend of financial conditions of local governments for a nationwide sample. We start by introducing the sample and the sample selection.

The nationwide sample is dictated by the data availability of financial data. The data is collected as an input in Moody’s rating methodology. As such the sample covers predominantly local governments that are active participants in the municipal bond market. Ex-ante we would expect that this selection favors local governments that are large in terms of its population. We find that the median population size is 21,187 and the mean population size is 59,787. While the observed selection on size may compromise the overall representativeness for local governments, we suspect that we capture the economically most relevant local governments. In addition, we restrict the sample to those observations that have non-missing values for the unrestricted net position over operating expenditures both in 2007 and 2018. This allows us to make inter-temporal comparisons without concerns about composition effects. We obtain a total sample of 1,803 local governments across the United States for which we tabulate summary statistics in Table A.2. We further show the geographic distribution in Figure SA.1.

The financial conditions for the nationwide sample show a deterioration over time. We present histograms for the two aforementioned financial indicators for the years 2007 and 2018. Figure 2a overlays the histograms of the unrestricted net position for the years 2007 and 2018. While in 2007 the distribution is centered and fairly symmetric around zero, the distribution shifts markedly to the left in 2018. Furthermore, the unrestricted net position shows a long and fat left tail. Concretely, the median decreases from 28.40% to -18.97% and the 5% percentile of the unrestricted net position over operating revenue

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11 Figure A.1 in the Appendix shows a schematic balance sheet and Table A.1 presents summary statistics of the most salient items in the national sample.

12 The median population is 1,030 and the mean population is 7,393 in the Census of Government in 2017. We have to conduct further validations of those numbers as the city with the smallest population is currently listed with zero population. While the anticipated sample selection towards larger local governments is present, we consider the median population of 21,187 as modest.
distribution decreases from -25.02% to -190.62%. At the very left tail the unrestricted net position is about 5 times its annual revenues in 2018. The observation about deteriorating financial conditions is not specific to the unrestricted net position. Figure 2b shows the two histograms for total debt over full value. Between 2007 and 2018 the median decreases from -0.49% to -1.99% and the 5% percentile of the total debt over full value distribution decreases from -3.37% to -8.86%.

Next, we show the discrepancy between the general fund balance—following fund accounting—and the unrestricted net position—based on accrual accounting. Figure 3a plots the general fund balance over operating revenues against the unrestricted net position over operating revenues. While only 5/1,803 (0.27%) local governments operate with a negative general fund balance, 1,099/1,803 (60.95%) operate with a negative unrestricted net position at the end of fiscal year 2018.

Further, we show the strong relationship of the unrestricted net position to legacy obligations. Figure 3b plots the unrestricted net position over operating revenues against net pension and net OPEB liabilities over operating revenues.

Lastly, we investigate to what extent the financial indicators from the financial reports align with market signals in the municipal bond market. We find that the unrestricted net

Figure 2: Financial Conditions Indicators

Notes: Panel (a) shows the unrestricted net position as a share of the the general fund total revenue in 2007 (green) and 2018 (transparent). Panel (b) plots the total liabilities (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over full value. The plot follows the convention that liabilities are expressed with a negative sign. Data is obtained from Moody’s Investor Service.
Panel (a) plots the budget balance of the general fund (end of fiscal year) and the unrestricted net position over operating revenues. Panel (b) plots the unfunded OPEB and pension liability and the unrestricted net position over operating revenues. The size of the circle corresponds to the total operating revenues of the corresponding local government. The sample contains all local governments for which the unrestricted net position is available in both 2007 and 2018. Data is obtained from Moody’s Investor Services.

position over operating revenues and total debt over the full value—is associated strongly with municipal bond spreads as shown in Figure 4a. The cross-sectional variation could in principle reflect relative differences in default risk and/or liquidity risk. Schwert (2017) argues that default risk accounts for 74% to 84% of the average yield spread after adjusting for the tax-exempt status. Thus, we interpret the cross-sectional variation of yields to reflect the difference in default risk rather than a compensation of liquidity risk.

Figure 3: Budget Balance and Liability Composition in 2018

Notes: Panel (a) plots the budget balance of the general fund (end of fiscal year) and the unrestricted net position over operating revenues. Panel (b) plots the unfunded OPEB and pension liability and the unrestricted net position over operating revenues. The size of the circle corresponds to the total operating revenues of the corresponding local government. The sample contains all local governments for which the unrestricted net position is available in both 2007 and 2018. Data is obtained from Moody’s Investor Services.
Panel (a) plots the relationship between the GZ Spread (Gilchrist and Zakrašek, 2012), a duration matched yield spread, for issuances with maturity of over one year at issuance, tax-exempt status, and classified as full general obligation and the unrestricted net position. Panel (b) plots the relationship between the GZ Spread for issuances with maturity of over one year at issuance, tax-exempt status, and classified as full general obligation and the total liability (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over the full value. Data on municipalities’ ACFRs is obtained from Moody’s Investor Services and primary bond issuance data is from Mergent Municipal Bond Database. All plots are binscatters with 30 quantiles.

### 3.3 Census Certainty Sample

For the remainder of the paper we focus on the so-called census certainty sample which consists of 388 local governments across the United States.\(^{13}\) This is the primary sample to estimate the governmental sector in BEA NIPA. Further, to the best of our knowledge, this is the broadest sample for which long time series on revenue and expenditures are available in the United States. First, we obtain additional information on book values from the Moody’s Investors Service Data and further manually collect book value information from ACFRs if available. The geographic location of the census certainty sample is exhibited in Figure A.2 and the summary statistics in Table A.3.

In terms of its fiscal position, the local governments in the census certainty sample show great similarity with those in the broader national sample. The median of the unrestricted net position over operating revenues in 2018 is -29.32% as opposed to -18.97%.

\(^{13}\)The intersection of local governments which are both in the census certainty sample and have book value information and other market based information available is 388. The full Census certainty sample consists of 622 entities. We hand collect data to get the best possible coverage.
the 25-th percentile is -81.11% as opposed to -84.62% and the 75-th percentile is 4.02% as opposed to 22.08%. We find this similarity also for the alternative fiscal indicator, that is, the total liability over full value. In 2018, the median is -3.09% in the census certainty sample compared to -2.34% in the broader national sample. Similarly to the national sample, about 23.1% of local governments operate with a negative net position and about 71.1% operate with a negative unrestricted net position. Overall, the census certainty sample shows similar characteristics to the much broader national sample. This is re-assuring since data limitations will limit us to this sample henceforth.

The similarity of the sample is also reflected in the municipal bond market. We repeat the analysis from above for the census sample in the primary and secondary bond market. Figure SA.2 shows the relationship between the unrestricted net position over operating revenues and the duration matched spread in the municipal bond market. One of the limitations in this analysis is that the sample is selected by the issuer’s activity in the municipal bond market. We can only include an observation if there was at least one issuance at the municipal bond market for the primary market and if there was at least one trade in the bond for the secondary market analysis. Reassuringly, the slope that we find the primary and secondary market brackets the slope of the national sample.

We find similar results, as presented above, for the total liabilities over full value. Figure SA.3 shows the association between the total liabilities over full value in the primary and secondary market. While the sample in the primary and secondary market once again differs due to the availability of transactions in the respective market, we find slopes that are remarkably similar to each other. Furthermore, the slope is of similar magnitude as for the broader national sample. This re-affirms once more the similarity of these two samples in terms of its financial characteristics.

We now proceed to estimate market valuations for these 388 local governments. The challenge here is valuing the revenue and expenditure streams of each local government by taking into account their exposure to different factors in the economy. We achieve this by estimating an exponentially affine asset pricing model with a rich set of state variables. Apart from showing that our independently estimated market valuations positively track book valuations, while also capturing additional variation not present in book valuations, the exercise allows us to estimate the implicit insurance provided by higher governments through an option pricing exercise.
4 Pricing the Components of a Local Government Balance Sheet

In this section, we discuss the main components of a local government’s balance sheet. In contrast to the discussion in the previous section in which we considered the book values as accounted by governmental accounting standards, we focus on the market values of the main components of the balance sheet. The valuation of assets and liabilities which is consistent with market prices provides a more forward-looking evaluation of local governments’ financial position.

In a first step, we are decomposing the balance sheet into its main components by starting with the basic accounting identity for equity.

\[ \textbf{Equity} = \textbf{Assets} - \textbf{Liabilities} \]

There is some debate about the assets of local governments. We follow the convention in the rating process which assumes that local governments cannot liquidate their capital assets.\(^{14}\) Thus, we are decomposing the assets into the present value of the revenue stream and current cash and cash equivalents which can be liquidated at short notice.

\[ \textbf{Assets} = \text{PV(Revenues)} + \text{Cash} \]

We can further unpack the liabilities in its components.

\[ \textbf{Liabilities} = \text{PV(Expenditures)} + \text{PV(PensionObligations)} + \text{PV(OPEB)} + \text{PV(Debt)} \]

The liabilities include the present value of expenditures plus the value of pension obligations and OPEB obligations, and the present value of outstanding debt. The full equation

\(^{14}\)While we have witnessed transfers of assets into the fiduciary funds in the last decade, we are unaware of liquidations of public assets to overcome financial distress. Bankruptcy chapter 9 is significantly different from chapter 11 as there is no provision in the law for liquidation of assets of the municipality and distribution of the proceeds to creditors. Such a provision would violate the Tenth Amendment to the Constitution and the reservation to the states of sovereignty over their internal affairs (United States Courts, 2023).
Equity = \text{PV(Revenues)} + \text{Cash} - \text{PV(Expenditures)} - \text{PV(PensionObligations)} - \text{PV(OPEB)} - \text{PV(Debt)}

(1)

Equation (1) constitutes the main equation for the determination of the market value of the equity position. It remains to show how we calculate each item of the right hand side of Equation (1). Subsection 4.1 will discuss the re-pricing of the market value of pension and OPEB obligations and of outstanding debt obligations. Section 5 introduces the model that we use to price the present value of revenues and expenditures.

4.1 Re-pricing of Balance Sheet Components

We are re-pricing pension obligations, other post employment benefits (OPEB) and long-term debt following some of the seminal papers in the literature.

**Long-term Debt**  Long-term debt are valued in accordance to the market’s expectation of the default probability as exhibited by the credit spread over treasuries (after accounting for the tax exemption when applicable).

\[ MV_{LTDebt} = BV_{LTDebt} \exp(-cs_\tau \tau) \]

(2)

where \( \tau \) is the duration of the overall long-term debt portfolio of the local government and \( cs_\tau \) the corresponding credit spread.

**Pension Obligations**  Pension are valued as if they constitute a risk free liability in accordance with some of the seminal papers in the literature (Giesecke and Rauh, 2022; Novy-Marx and Rauh, 2009, 2011a; Brown and Wilcox, 2009; Lucas and Zeldes, 2006).

\[ MV_{NetPensionLiability} = BV_{NetPensionLiability}[1 + Duration_{NPL}(y_{Pension} - y_{try}) + \frac{1}{2} Convexity_{NPL}(y_{Pension} - y_{try})^2] \]

(3)
where $y_{\text{Pension}}$ is the actuarially assumed discount rate and $y_{\text{try}}$ is the duration matched treasury yield.

**Other Post-Employment Benefits** While there is some debate whether other post-employment benefits enjoy the same protections as pension benefits, we value the liability consistent with the pension liabilities (Pozen and Rauh, 2015; Joffe, 2021).

\[
MV_{\text{NetOPEB}} = BV_{\text{NetOPEB}}[1 + Duration_{\text{NPL}}(y_{\text{OPEB}} - y_{\text{try}}) + \frac{1}{2}Convexity_{\text{NPL}}(y_{\text{OPEB}} - y_{\text{try}})^2]
\]  

(4)

where $y_{\text{OPEB}}$ is the actuarially assumed discount rate and $y_{\text{try}}$ is the duration matched treasury yield.
5  Present Value of Revenues and Expenditures

In Section 4.1 we discussed the re-pricing of pension and OPEB obligations and long-term debt. The remaining two components are the present value of revenues and expenditures. Unfortunately, revenue and expenditure claims are not traded in the market and thus no market prices are available. Furthermore, revenue and expenditure claims may potentially have risky payoffs which results in a price adjustment from a risk-free annuity. We overcome these limitations by estimating a stochastic discount factor that prices a broad set of assets in the economy. The existence of a (strictly positive) stochastic discount factor allows us to price the revenue and expenditure claims such that there are no arbitrage opportunities between traded assets and non-traded claims.

Our model consists of three components: (i) a VAR that governs the evolution of the state vector \( z_t \); (ii) an exponentially affine asset pricing model that describes the stochastic discount factor \( M_{t+1}^\delta \); (iii) a spanning argument that allows us to price variables outside the state vector, in particular revenue and expenditure claims of local governments.

5.1 Evolution of state variables

There is a \( N \times 1 \) vector \( z \) of state variables that follows a first order VAR with Gaussian error:

\[
z_t = \Psi z_{t-1} + u_t = \Psi z_{t-1} + \Sigma_1^{1/2} \varepsilon_t
\]

where \( \Psi \) is a \( N \times N \) companion matrix, \( u_t \) is a Gaussian error \( u_t \sim i.i.d. \mathcal{N}(0, \Sigma) \), the Cholesky decomposition of the covariance matrix gives us the lower triangular matrix \( \Sigma_1^{1/2} \), with structural shocks \( \varepsilon_t \sim i.i.d. \mathcal{N}(0, \Sigma) \). The vector \( z \) is demeaned by the sample averages of each individual element.

We include a rich set of state variables: lagged inflation, GDP growth, short yield, the 5-1 year yield spread, the stock market price-dividend ratio and dividend growth. We include both levels and growth of federal taxes and spending following Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019). Importantly, we add the municipal credit spread to the VAR. The municipal credit spread is defined as the difference between the 10-year reference municipal yield from Bondbuyer and the 10 year Treasury bond yield. Table 1 provides a list of these variables along with their sample means.
The inclusion of spending and taxes in the state vector means that we assume the federal government commits to a policy that is affine in the state vector. Including the levels of these two variables along with their growth rates means that we capture the idea of automatic stabilizers since any deviation in growth rates leads to a reversion to the long run level relative to GDP. The credit spread of the reference municipal yield captures an important aggregate component of the muni bond market.

<table>
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<tr>
<th>Position</th>
<th>Variable</th>
<th>Variable Mean</th>
<th>Sample Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \pi_t )</td>
<td>( \pi_0 )</td>
<td>0.03108</td>
</tr>
<tr>
<td>1</td>
<td>( x_t )</td>
<td>( x_0 )</td>
<td>0.029745</td>
</tr>
<tr>
<td>2</td>
<td>( y(1)^S_t )</td>
<td>( y(1)^S_0 )</td>
<td>0.04329</td>
</tr>
<tr>
<td>3</td>
<td>( y spr^S_t )</td>
<td>( y spr^S_0 )</td>
<td>0.005838</td>
</tr>
<tr>
<td>4</td>
<td>( pd_t )</td>
<td>( pd_0 )</td>
<td>3.528392</td>
</tr>
<tr>
<td>5</td>
<td>( \Delta d_t )</td>
<td>( \Delta d_0 )</td>
<td>0.060559</td>
</tr>
<tr>
<td>6</td>
<td>( \Delta \log \tau_t )</td>
<td>( \Delta \log \tau_0 )</td>
<td>-0.006712</td>
</tr>
<tr>
<td>7</td>
<td>( \log \tau_t )</td>
<td>( \log \tau_0 )</td>
<td>-2.236345</td>
</tr>
<tr>
<td>8</td>
<td>( \Delta \log g_t )</td>
<td>( \Delta \log g_0 )</td>
<td>0.001887</td>
</tr>
<tr>
<td>9</td>
<td>( \log g_t )</td>
<td>( \log g_0 )</td>
<td>-2.148222</td>
</tr>
<tr>
<td>10</td>
<td>( \Delta \log d_t )</td>
<td>( \Delta \log d_0 )</td>
<td>0.003952</td>
</tr>
<tr>
<td>11</td>
<td>( \log d_t )</td>
<td>( \log d_0 )</td>
<td>-1.042491</td>
</tr>
<tr>
<td>12</td>
<td>( cs_t )</td>
<td>( cs_0 )</td>
<td>-0.003064</td>
</tr>
</tbody>
</table>

Table 1: State Variables

Estimation We estimate the VAR using OLS. The point estimates for \( \Psi \) are reported in Table 2. The point estimates for the Cholesky decomposition \( \Sigma^2 \) is reported in Table 3.

| \( \pi_{t-1} \) | \( x_{t-1} \) | \( y(1)^S_{t-1} \) | \( y spr^S_{t-1} \) | \( pd_{t-1} \) | \( \Delta d_{t-1} \) | \( \Delta \log \tau_{t-1} \) | \( \Delta \log \tau_{t-1} \) | \( \Delta \log g_{t-1} \) | \( \Delta \log d_{t-1} \) | \( \log d_{t-1} \) | \( cs_{t-1} \) |
|----------------|--------------|------------------|------------------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---------|
| 0.67          | -0.14        | -0.11            | -0.21            | -0.00     | 0.01           | 0.03            | -0.05          | -0.04          | -0.01          | -0.05          | -0.02          | -0.29   |
| 0.08          | 0.08         | 0.57             | -0.12            | -0.00     | 0.04           | -0.01           | -0.03          | -0.03          | 0.00           | -0.00          | -0.55          |         |
| -0.03         | -0.09        | 0.08             | 0.40             | -0.01     | 0.02           | 0.01            | 0.02           | 0.01           | 0.02           | 0.00           | 0.30           |         |
| -0.22         | -1.03        | -0.31            | -3.05            | 0.69      | -0.21          | -0.35           | 0.05           | 0.08           | -0.28          | -0.07          | 0.17           | 3.90    |
| 1.69          | -0.01        | -0.91            | -1.36            | 0.08      | 0.29           | -0.11           | -0.30          | -0.20          | 0.15           | -0.13          | 0.20           | -1.42   |
| 1.67          | 0.31         | -0.22            | -3.08            | 0.17      | 0.07           | -0.01           | -0.55          | -0.18          | 0.35           | -0.09          | 0.20           | 1.80    |
| 1.67          | 0.31         | -0.22            | -3.08            | 0.17      | 0.07           | -0.01           | 0.45           | -0.18          | 0.35           | -0.09          | 0.20           | 1.80    |
| 0.00          | 0.00         | 0.12             | 0.45             | 0.04      | 0.02           | 0.00            | -0.01          | 0.00           | -0.01          | 0.00           | -0.02          | 0.45    |

Table 2: VAR Coefficients
5.2 Asset Pricing

We use an exponentially affine model (Duffie and Kan (1996)) to price the stochastic discount factor. The advantage of this model is that it only assumes no-arbitrage, prices bond yields well, as well as provides a reasonable equity risk premium. The nominal SDF is conditionally log-normal:

\[ m^s_{t+1} = -y^s_t(1) - \frac{1}{2} \Lambda^t \Lambda^t' \varepsilon_{t+1} \]  

where \( m^s_{t+1} = \log(M^s_{t+1}) \) the short rate is \( y^s_t(1) \) and the \( \Lambda_t \) vector prices the sources of risk in the structural innovations \( \varepsilon_{t+1} \). Further, the \( \Lambda_t \) vector is expressed as the combination of an unconditional price of risk and a time varying component.

\[ \Lambda_t = \Lambda_0 + \Lambda_1 z_t \]

Here, \( \Lambda_1 \) is a \( N \times N \) matrix that provides the time variation in risk premia while \( \Lambda_0 \) is the average price of risk in a \( N \times 1 \) vector.\(^{15}\) In what follows, the selector vector is defined as \( \varepsilon(\cdot) \).

**Nominal Bond Pricing**

The nominal yields are given by:

\[ y^s(h) = -A^s(h) \frac{h}{h} - B^s(h)' z_t \]  

\(^{15}\)This approach has been used more recently in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019) and Gupta and Van Nieuwerburgh (2021), among others.
Since we have the 5-1 yield spread, we can use it to obtain the following moment restrictions:

\[ e_{y1}^{'} + e_{yspr}^{'} = -\frac{1}{5}B_{5}^{s} \]  
\[ y_{0,1}^{s} + yspr_{0} = -\frac{1}{5}A_{5}^{s} \]  

where \( \Psi = \Psi - \Sigma^{\frac{1}{2}} A_{1} \) is the risk neutral companion matrix. Then we use equation (3) to fit yields of 2, 10, 20, 30 years from the data.

**Real Bond Pricing**

The real bond yields are also affine in the state vector:

\[ y_{t}(h) = -\frac{A(h)}{h} - \frac{B(h)^{'}z_{t}}{h} \]  

We match data for real bond yields for 5, 7, 10, 20 and 30 years using the above expression.

**Equity Pricing**

The log price-dividend ratios on dividend strips are affine in the state vector:

\[ pd_{t}^{m}(h) = A^{m}(h) + (B^{m}(h))^{'}z_{t} \]  

We take the first 3600 dividend strip horizons to match each date’s ratio with the data:

\[ \exp(pd + (e_{pd})^{'}z_{t}) = \sum_{h=0}^{\infty} \exp(A^{m}(h) + (B^{m}(h))^{'}z_{t}) \]  

where \( pd \) is the mean log price dividend ratio in the data. We also match the equity risk premium.

**Estimation** We estimate the model by matching federal government bond prices and stock prices in the data with predictions from the model. Appendix 7 contains the relevant moment conditions. The estimated \( \Lambda_{0} \) and \( \Lambda_{1} \) are provided in Appendix B.7.3.

As can be seen in Figure A.4, we get very good fit for the nominal bond yields at various maturities. In Figure A.5, we also get a decent fit for real bond yields.
5.3 Pricing Local Government Claims

A local government claim $W$ can be decomposed into strips. Each strip is defined by its horizon $j$ at time $t$, that pays off $W_{t+j}$ at time $t+j$ and nothing otherwise. There will be a relevant holding period return $R_{t,t+j}^{w,j}$ on these strips. The local government component is not part of the state vector $z_t$. The nominal growth rate of this component is

$$\Delta \log W_{t+1} = \log \frac{W_{t+1}}{W_t} = \log \left[ \frac{W_{t+1}}{W_t} \times \frac{Y_{t+1}}{Y_t} \times \frac{Y_t}{Y_{t+1}} \right]$$

Define $w = W/Y$, where $Y$ is the GDP. $x$ is the log GDP growth rate, $\pi$ is the log inflation.

$$\Delta \log W_{t+1} = \Delta \log w_{t+1} + x_{t+1} + \pi_{t+1}$$

We postulate that the growth rate in local government claim is spanned by the state vector

$$\Delta \log w_{t+1} = w_0 + T'z_{t+1} + U'\eta_{t+1}$$

where $w_0$ is the average value of $w$ in the data, $\eta_{t+1}$ is an added shock vector that is orthogonal to those included in $\varepsilon_{t+1}$. It can then be shown (see Appendix B.7.4) that the valuation ratio of the government claim is

$$\sum_{h=0}^{\infty} \exp(A^w(h+1) + B^w(h+1)'z_t) \quad (13)$$

**Pricing:** We price revenue and expenditure claims by first projecting its growth rate onto the state variables of our model. The validity of the pricing rests on the assumption that all pricing relevant information is spanned by the state variables. Our model includes 13 state variables, including the municipal credit spread, that captures several sources of pricing relevant risk. The spanning assumption is common in many cross-sectional asset pricing models. We use equation 13 to estimate the valuation ratio of the claim at each time $t$. 

26
6 Market Valuation

One of the limitations of the analysis which is based on accounting values is that accounting values are mostly backward looking. For instance, capital projects are often valued at the purchasing cost minus depreciation rather than at the present value of expected generated cash-flow. Further, some of the revenue potential for local governments is not tied to a specific asset; instead it is the result of the privilege to raise taxes. While we think that accounting values carry some merit, we acknowledge its limitations. Thus, we complement the analysis from Section 3 with a market based valuation of local governments’ equity.

One of the challenges is that market prices for revenue and expenditure claims are not directly observable in the market. We thus estimate a stochastic discount factor as detailed in Section 5. The stochastic discount factor prices a large set of assets in the economy as shown in Figure A.4 and A.5. This includes municipal debt securities as shown by the close fit with one of the main indices in the municipal bond market as shown in Figure A.3. The existence of a stochastic discount factor is sufficient to price those claims consistently with other assets for which prices are observable (Cochrane, 2009).

6.1 Cross-Sectional Exposure

In the absence of risk the value of a claim is given by its present value discounted at the risk free rate. While this is a convenient benchmark, it is equally unrealistic. Local governments’ sales and excise taxes are mechanically tied to the turnover of goods and services in the economy which tend to be pro-cyclical. Similarly, property taxes are to some extent related to the valuations in the local housing market.\footnote{The extent to which the property tax revenues follow the valuation depends from state to state due to the autonomy to set property taxes (Reschovsky, 2019). For instance, Giesecke and Mateen (2022) show that local government offset the decline in house prices by increasing the property tax rate in Connecticut. As a result, the effect on total property tax revenues is muted.} Hence, local governments’ receipts are exposed to aggregate risk which should be reflected in the asset’s valuation. An analogous argument applies to government expenditures. While local governments have much less discretion about the timing of expenditures due to the balanced budget requirement than the federal government, we observe a counter-cyclical pattern
in expenditures.

For illustrative purposes let us consider a one factor model first in which the real GDP growth rate is the only risk factor in the economy. While the real GDP growth rate is most likely not the only risk factor, it tends to be one of them in canonical asset pricing models.

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<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.563***</td>
</tr>
<tr>
<td></td>
<td>(0.0340)</td>
<td>(0.0844)</td>
</tr>
<tr>
<td>Share property tax rate</td>
<td>0.00207</td>
<td></td>
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<tr>
<td></td>
<td>(0.00201)</td>
<td></td>
</tr>
<tr>
<td>Real GDP growth rate × Share property tax rate</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.114)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.002</td>
</tr>
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</tr>
<tr>
<td>Observations</td>
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</tr>
</tbody>
</table>

Table 4: Risk Exposure

Notes: The tables shows the estimates of the following specification. Column (1) $\Delta \ln \text{OwnSource}_{it+1} = \alpha_i \text{time}_{t+1} + \beta \Delta \ln \text{OrGDP}_{it+1} + \epsilon_{it+1}$ and column (2) Column (1) $\Delta \ln \text{OwnSource}_{it+1} = \alpha_i \text{time}_{t+1} + \beta_1 \text{Sharepropertytax}_i + \beta_2 \Delta \ln \text{OrGDP}_{it+1} + \beta_3 \Delta \ln \text{OrGDP}_{it+1} \times \text{Sharepropertytax}_i + \epsilon_{it+1}$, where $\text{Sharepropertytax}_i$ is the average share of property taxes as of total own source revenues over the sample horizon for local government $i$ and $\text{time}_{t+1}$ is a deterministic time trend.

Column 1 of Table 4 shows that the de-trended growth rate of own source receipts shows exposure to the real GDP growth rate. Importantly, there is substantial cross-sectional heterogeneity in the exposure; column (2) explores the heterogeneity with respect to the share of own source receipts that originate from property taxes. Consistent with the hypothesis above, local governments that receive a large share from property taxes are relatively less exposed to the business cycle fluctuations than local governments with other sources of revenue.

Alternatively, we can estimate the exposure for each local government separately. Figure 5a shows the full distribution of the estimated exposure in the cross-section of local governments. We find a large range of cross-sectional exposure estimates with a mode slightly above zero. The relationship between this individually estimated exposure and the mean share of property taxes of own source revenues is shown in Figure 5b.
Notes: The histogram in Panel (a) shows the cross-sectional exposure of $\Delta \ln \text{OwnSourceRevenue}_{it+1}$ to real GDP growth rate $\Delta \ln r_{GDP}$. Specifically, it tabulates the estimates of the exposure $\hat{\beta}_i$ from the following specification: $\Delta \ln \text{OwnSourceRevenue}_{it+1} = \alpha_{time_t+1} + \beta_i \Delta \ln r_{GDP} + \epsilon_{it+1}$ which is estimated for each local government $i$ separately and where $time_t$ is a deterministic time trend. Panel (b) shows a bincscatter with 30 bins of the cross-sectional relationship between the exposure estimates $\hat{\beta}_i$ and the average share of property taxes as of total own source revenues over the sample horizon for each local government $i$.

### 6.2 Valuation Ratios

In the previous Subsection 6.1 we showed the exposure to a single risk factor for illustrative purposes and to create intuition for the origin of some of the heterogeneities. For the pricing we use the full set of state variables as detailed in Section 5. Ultimately, the valuation ratio is given by Equation 13 which is exponentially affine in the exposure to the state variables and the corresponding risk premia. Figure 6 summarizes the estimation results by tabulating the valuation ratios for revenues and expenditures. Interestingly, and to some extent mirroring the insights from Subsection 6.1, we find substantial heterogeneity across local governments in terms of their valuation ratios.

To interpret the exact numbers of valuation ratios we find it is useful to start from a risk-free benchmark. If revenues were deterministic and just for the sake of the argument, assuming that the relevant risk-free interest rate is 3% (for reference, if we take the 30 year treasury which hovered around 3% during this sample period), one would expected a valuation ratio of $1/0.03 = 33.3$. Estimated valuation ratios of revenues are smaller–for some much smaller–than this risk-free benchmark which points to pro-cyclical revenues of local
governments. This pattern of revenues mirrors the pro-cyclicality of federal government revenues as reported in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019).

6.3 Book values vs. Market values

The market valuation ratios allow us to compute the market value of equity. In particular, we multiply the contemporaneous revenues and expenditures with the corresponding valuation ratio and deduct the value of the outstanding liabilities.\footnote{We avoid double counting of cash-flows to debt, other post employment benefits and pension by deducting the interest expenses, the pension and OPEB contributions from current expenditures.} This provides us with a measure of market equity for each local government. Table 7 shows the cross-sectional relationship between the market value of equity and the two accounting measure of market values from the ACFRs, that is, the net position and the unrestricted net position. For comparability across local governments all values are normalized by the operating revenues. We find a positive relationship between the book values and the market values for the net position and a stronger positive relationship for the unrestricted net position. The stronger relationship for the unrestricted net position is not surprising since the inclusion of capital assets into the net position adds additional variation which is not necessarily reflected in its revenue generating potential.
6.4 Equity Value of Local Governments and Role of Implicit Insurance

Is the market value of equity really negative for certain local governments? If that were the case, it would mean that these local governments are insolvent, and debt markets should not be issuing debt to these entities. But we do not see this. In fact, municipal yields of highly indebted governments offer limited risk compensation for a potential default.

In this section, we offer a possible resolution of the contrasting evidence. We introduce the idea that the intervention by federal and state governments constitutes a “put” option, whose value is taken into account by bond investors.\(^\text{18}\) Interventions from higher governments take many different forms. First, there may be Chapter 9 bankruptcy proceedings. A useful case study here is the bankruptcy of Detroit, MI, in 2013. The city could not generate revenues to match their expenses for, most prominently, pension costs (Giesecke, 2022). The city filed for chapter 9 bankruptcy a few months after a state government report declared the city to be “clearly insolvent on a cash-flow basis.” Subsequently, the state legislature provided about $200 million in lump-sum support for the pension

\(^{18}\)Local governments also have taxing power, and with the caveat that any increase in taxes may lower the tax base, local governments may offer implicit insurance to any outstanding debt through their taxing power. However, by using 40 years of data of the stochastic properties of local government expenditure, we are already capturing these patterns, including their covariance with the stochastic discount factor.
system, with additional funding slated over the next 20 years, for a total of $350 million. However, bankruptcies are rare – there have been only 37 such cases since the year 2000 (Duffy and Giesecke, 2023). Second, there are many examples of “silent interventions.” These are preemptive interventions by higher governments, typically state governments, who provide “tit for tat” support based on laid-out conditions. For example, Milwaukee, WI, received a concession from the state government of Wisconsin to obtain a larger share of the state’s sales tax revenues and to levy their own local sales tax. In return, they were required to close their local pension fund to any new employees. Third, there are relief packages that are typically provided by the federal government. The most recent and salient example is the support during the COVID-19 pandemic, when the CARES Act, FFCRA, RRA, and ARPA provided about $415bn to local governments (Clemens, Hoxie, and Veuger, 2022). A large sum was also disbursed during the Great Financial Crisis through the ARRA, Build America Bond, and federal infrastructure grants. Even though the actual process in the above cases are complicated, at times with multiple rounds of negotiations, our approach below captures two important aspects of the proceedings. First, government intervention is triggered when the ability to service cash-outflows is compromised. Second, the government intervention can occur over several years.

We define the minimum value of insurance that is necessary to validate the expectation of market participants in the municipal bond market as the amount that is necessary to cover the negative market equity position as shown in Equation 14.

\[
IS = \max\{0, PV(\text{Expenditures}) + PV(\text{PensionObligations}) + PV(\text{OPEB}) + PV(\text{Debt}) - PV(\text{Revenues}) - \text{Cash}\}
\]

(14)

Naturally, the actual insurance could be higher than this; as such our definition is a lower bound on the actual insurance. This approach assumes that there is no mispricing by the municipal bond market and that a simple accounting of visible components of cash flow does not consider the implicit insurance value of governmental support. Start with debt. We take debt at its face value and we assume that the yields represent the expectations of market participants as to the default probability of the municipality, all factors considered, including insurance. On the pensions side, there are constitutional safeguards protecting the claims of pensioners, with court judgements ruling that these claims are
senior to bond holder debt (see Spiotto (2013), Boyer (2018)). We discount these claims by the risk free rate, thereby following a rich literature investigating the riskiness of pension payouts (see Novy-Marx and Rauh (2011a)) that concludes that pension payouts are unrelated to the business cycle and therefore should be discounted at the risk free rate.

Assuming that the federal and state government are providing this insurance, it would be interesting to estimate the value of this “put” option, and to see if it can adequately explain negative values of market equity. In the absence of markets providing options on municipalities, it is difficult to back out the value of the option as in Kelly, Lustig, and Van Nieuwerburgh (2016). Instead, we simulate the future evolution of revenues and expenditures for each municipality in our sample, using historical growth rates of these flows. To be precise, we obtain projection coefficients from a regression of municipality revenue and expenditure growth rates on the state vector using data for each municipality for the past 40 years. We then generate 1000 trajectories for each municipality by drawing random errors and using (i) the companion matrix and Cholesky matrix for evolution of the state vector; (ii) the projection coefficients for the evolution of the revenue and expenditure flows. We use the last year in our data set to set the starting value of each variable. In each period within each trajectory, we assume that the municipality receives an option payout if the deficit of the municipality (defined as the difference between expenditures and revenues) exceeds the municipality’s revenues in that period times a constant. This constant, which we call the intervention threshold, $\omega$, simply allows for the option to not necessarily pay out when expenditures exactly exceed revenues.

Therefore, we have the cash flow from the option which is non-zero for a period if the deficit exceeds the threshold. We use the risk free rate in that period and trajectory to calculate the present value of the cash flow. We use the following estimator to calculate the price of the option, $\mathbb{E}_T[M_{T+k}X_{T+k}(\omega)]$:

$$P_{T,T+k}(\omega) = \frac{1}{1000} \sum_{i=1}^{1000} M_{T,T+k}^i X_{T+k}^i(\omega)$$

where $T$ is the starting period, $i$ is an index for the trajectory, $X_{T+k}^i$ is the option payout
in time period \( T + k \), \( M^i_{T,T+k}(\omega) \) is the SDF from period \( T \) to period \( T + k \). \(^{19}\)

Then the total insurance price (over \( K \) periods) is:

\[
P_T(\omega) = \sum_{k=1}^{K} P_{T,T+k}(\omega)
\]

The question of interest here is how this option price compares with the market value of equity in the previous section. According to historical data and calculated projection coefficients, the growth rates of revenues and expenditures are often different from each other, and are both greater than zero. This means that municipalities with higher rates of revenue growth versus expenditure growth are always healthy in the long run, with the option cash flow being close to zero. There are, however, many other municipalities where expenditure growth is close to or even greater than revenue growth. In the latter case, once the level of expenditure exceeds revenue growth by the threshold, it will continue to stay above it and the option will be constantly in the money. This is because we do not assume any restructuring of government finances after option payout(s). To capture this effect, we cap the number of periods the option will payout before the municipality’s finances are restructured so that there are no deficit violations in the infinite future. We then study the effect of varying the length of the periods over which the option would provide cash flow for deficits that exceed the threshold.

We take the example of Birmingham, Alabama. Figure 8 plots the relationship between Birmingham’s option price as the intervention period increases from 1,2,5,10 to 20 years. On the same plot, the negative of the market equity of Birmingham is plotted. We see that the option price increases with the duration of the intervention period. This is because, as the option pays out for multiple time periods the value of the option is higher. Therefore, we see a monotonic increase in option value with the intervention period. We also find that the intervention needs to be for at least 5 years to justify the market value of equity for Birmingham, calculated in the previous section. This suggests that the market expects higher levels of government to not only intervene to support local government finances, but also that the market expects this intervention to last for several years.

\(^{19}\)In the analysis here, to fix ideas, we use the risk free rate instead of the SDF. This allows us to separate out the cash flow evolution of the claims from the reweighing of these claims depending on the period marginal utility. Our next step will be to use the SDF for the discounting. This we obtain from the evolution of the state vector and the estimated prices of risk.
We assess the value of the insurance option more systematically in our sample of 388 city governments. Concretely, we ask for what fraction of the sample, the option value is sufficiently large to cover the previously identified negative market equity position. Table 5 tabulates the results for intervention horizons of 1, 5 and 10 years. We find that for an intervention horizon of 1 year the share is 35.1%. That means, for about 64.9% of city government with negative market equity the option value is not sufficiently large enough. The share increases to 42.0% and 45.7% with an increase in the intervention period to 5 and 10 years, respectively.

The insurance option is designed to cover future cash-flow deficits but, in its current design, leaves the legacy liability unaddressed. We find that even with a very long intervention period, for approximately 20% of the city governments in our sample the value
of the option remains insufficient to cover the municipalities’ negative market value of equity. Alternative designs of the insurance option remain subject of future research.
7 Conclusion

Using a novel data set on the fiscal position of municipalities across the US we document the deterioration of municipalities’ fiscal positions. Besides the overall deterioration, a substantial share of municipalities operate with a negative net position—akin to a negative book equity position in the corporate context. Book valuations may provide an incomplete assessment of local government’s solvency as it follows a rigid set of governmental accounting standards which are predominantly backward looking. Thus, we assess the market value of equity by pricing the main components of local governments’ balance sheet. We do this by estimating an exponentially affine term-structure model that prices a broad array of assets in the economy. We use the model to price a broad cross-section of municipal claims. To the best of our knowledge, this is the first paper to be able to price such a broad array of assets. By being able to provide a market value measure of municipalities equity, we show that book values are positively related to the market valuations which further supports the initial assessment of local governments’ fiscal position. Somewhat surprisingly, the municipal bond market discriminates the differences in the fiscal position only to a limited extent. We attribute the limited distinction in credit to the belief in implicit insurance by the state and federal government. We show that an insurance option with an intervention period of 10 years, is able to cover the negative equity position for approximately half of all city governments with negative equity. Our work raises important questions about the financial sustainability of local governments in the United States.
References


### Tables and Figures:

#### Figure A.1: Schematic Balance Sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash &amp; Invest.</td>
<td>Net Position</td>
</tr>
<tr>
<td>Capital Assets</td>
<td>LT Debt</td>
</tr>
<tr>
<td>Other Assets</td>
<td>Pensions</td>
</tr>
<tr>
<td></td>
<td>OPEB</td>
</tr>
<tr>
<td></td>
<td>Other Liabilities</td>
</tr>
</tbody>
</table>

#### Table A.1: National Sample - Balance Sheet 2018

<table>
<thead>
<tr>
<th>Assets</th>
<th>mean</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share Cash Investments</td>
<td>0.24</td>
<td>0.16</td>
<td>0.21</td>
<td>0.29</td>
<td>1,803</td>
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<td>Share Receivables</td>
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<tr>
<td>Share LT. Illiquid Assets</td>
<td>0.06</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
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<tr>
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</tr>
<tr>
<td>Liabilities</td>
<td>mean</td>
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<td>count</td>
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<td>0.17</td>
<td>1,803</td>
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<td>Share Net Pension</td>
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<td>0.11</td>
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<tr>
<td>Share Other Current Liabilities</td>
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<td>-0.05</td>
<td>-0.03</td>
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<tr>
<td>Share Other Non-Current Liabilities</td>
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<td>0.06</td>
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<td>Share Net Position</td>
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<td>0.18</td>
<td>0.46</td>
<td>0.65</td>
<td>1,803</td>
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</table>

**Notes:** The table tabulates the main asset and liability positions of the Statement of Net Asset Position in the ACFR of municipalities across the United States. The sample contains all local governments for which information on the unrestricted net position is available in 2007 and 2018. Data is obtained from Moody’s Investor Services.
### Table A.2: Summary Statistics National Sample

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Revenues 2018 (in '000)</td>
<td>161871.62</td>
<td>16231.00</td>
<td>36396.78</td>
<td>83926.94</td>
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</tr>
<tr>
<td>GF Balance as of Op. Rev 2007 (%)</td>
<td>25.94</td>
<td>11.89</td>
<td>20.58</td>
<td>34.74</td>
<td>1,802</td>
</tr>
<tr>
<td>GF Balance as of Op. Rev 2018 (%)</td>
<td>33.40</td>
<td>16.83</td>
<td>26.93</td>
<td>42.74</td>
<td>1,802</td>
</tr>
<tr>
<td>Total liability over EGL 2007 (%)</td>
<td>-1.12</td>
<td>-1.83</td>
<td>-0.98</td>
<td>-0.45</td>
<td>1,726</td>
</tr>
<tr>
<td>Total liability over EGL 2018 (%)</td>
<td>-3.22</td>
<td>-3.97</td>
<td>-2.34</td>
<td>-1.18</td>
<td>1,784</td>
</tr>
<tr>
<td>Δ Total liability over EGL 07-18 (%)</td>
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<td>-2.49</td>
<td>-1.10</td>
<td>-0.29</td>
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<tr>
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<tr>
<td>Fraction Negative Net Position 2018</td>
<td>0.15</td>
<td>0.00</td>
<td>0.00</td>
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<td>1,803</td>
</tr>
<tr>
<td>Net OPEB as of Op. Rev 2018 (%)</td>
<td>-34.89</td>
<td>-50.61</td>
<td>-11.96</td>
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<td>1,803</td>
</tr>
<tr>
<td>Population (Census 2010)</td>
<td>59435.05</td>
<td>10292.00</td>
<td>21193.00</td>
<td>46746.00</td>
<td>1,803</td>
</tr>
<tr>
<td>Median House Value (Census2010)</td>
<td>266039.45</td>
<td>135700.00</td>
<td>135700.00</td>
<td>330600.00</td>
<td>1,803</td>
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<tr>
<td>Per Capita Income (ACS 2010)</td>
<td>31609.13</td>
<td>22418.00</td>
<td>27941.00</td>
<td>36467.00</td>
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<tr>
<td>Share 65+ Age (Census2010)</td>
<td>0.14</td>
<td>0.11</td>
<td>0.14</td>
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<td>1,803</td>
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<tr>
<td>Share White (Census2010)</td>
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<td>0.87</td>
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<td>0.01</td>
<td>0.03</td>
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</tr>
<tr>
<td>Share Asian (Census2010)</td>
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<td>0.01</td>
<td>0.02</td>
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<td>1,803</td>
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<tr>
<td>Home Ownership (Census2010)</td>
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<td>0.56</td>
<td>0.66</td>
<td>0.78</td>
<td>1,803</td>
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</tbody>
</table>

**Notes:** The sample contains all local governments for which information on the unrestricted net position is available in 2007 and 2018. Counts of less than 1803 observations indicate missing data. The table follows the sign convention that liabilities are expressed as a negative values. Data is obtained from Moody’s Investor Services.

### Figure A.2: Census Certainty Sample

**Notes:** The sample contains all cities that are part of the Census certainty sample. It comprises a total of Shape files for local governments are self-constructed as described in the Data Appendix DA.7.2. Data is obtained from the Census of Governments, the Census Bureau and Moody’s Investor Services.
<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>count</th>
</tr>
</thead>
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<tr>
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<td>68132.50</td>
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<tr>
<td>Median House Value (Census 2010)</td>
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<td>114900.00</td>
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<td>Per Capita Income (ACS 2010)</td>
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<td>20906.00</td>
<td>24478.00</td>
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<td>621</td>
</tr>
<tr>
<td>Share 65+ Age (Census 2010)</td>
<td>0.13</td>
<td>0.11</td>
<td>0.13</td>
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<td>622</td>
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<tr>
<td>Share White (Census 2010)</td>
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<td>0.76</td>
<td>0.88</td>
<td>622</td>
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<tr>
<td>Share Black (Census 2010)</td>
<td>0.14</td>
<td>0.02</td>
<td>0.06</td>
<td>0.20</td>
<td>622</td>
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<tr>
<td>Share Asian (Census 2010)</td>
<td>0.04</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
<td>622</td>
</tr>
<tr>
<td>Home Ownership (Census 2010)</td>
<td>0.59</td>
<td>0.51</td>
<td>0.59</td>
<td>0.66</td>
<td>622</td>
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<tr>
<td>Fraction Negative Unr. Net. Pos. 2018</td>
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<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>560</td>
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<tr>
<td>Total Liabilities of Full Value 2017 (%)</td>
<td>-4.00</td>
<td>-4.72</td>
<td>-2.93</td>
<td>-1.57</td>
<td>530</td>
</tr>
<tr>
<td>Total Liabilities of Full Value 2018 (%)</td>
<td>-4.63</td>
<td>-5.30</td>
<td>-3.05</td>
<td>-1.70</td>
<td>530</td>
</tr>
<tr>
<td>Net Position as of Op. Rev 2017 (%)</td>
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<td>49.01</td>
<td>138.36</td>
<td>283.73</td>
<td>560</td>
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<tr>
<td>Long Term Debt as of Op. Rev 2017 (%)</td>
<td>-96.58</td>
<td>-126.57</td>
<td>-76.15</td>
<td>-42.07</td>
<td>567</td>
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**Table A.3: Summary Statistics Census Certainty Sample**

*Notes*: The table tabulates the summary statistics for the Census certainty sample. Counts of less than 622 observations indicate missing data. The table follows the sign convention that liabilities are expressed as a negative values. Data is obtained as described in Section 2.

![Municipal 10 Benchmark Yield](image)

**Figure A.3: SDF - Municipal Bond Yield Index**

*Notes*: The figure plots the model implied and the actual municipal bond yield. The municipal bond yield is the 10 year reference yield from BondBuyer for the time horizon 1977-2019. The data comes from Bloomberg.
Figure A.4: SDF - Nominal Yields

Notes: The figure plots the model implied and the data for the 1yr, 2yr, 10yr, 20yr, and 30yr treasury yield for the time horizon 1977-2019. The data comes from FRED and FRASER.
Figure A.5: SDF - Real Yields

Notes: The figure plots the model implied and the data for the 5yr, 7yr, 10yr, 20yr, and 30yr TIPS yield for the time horizon 2010-2019. The data comes from FRED.
Appendix B: Moment Conditions

Call $\Psi$ the companion matrix, $\Sigma$ the covariance matrix, and $\Sigma^{\frac{1}{2}}$ the cholesky decomposition of the covariance matrix. The selector vector is defined as $e_{(.)}$.

B.7.1 Moment Conditions

Nominal Bond Pricing

The nominal yields are given by:

$$y^{s}_t(h) = -\frac{A^s(h)}{h} - \frac{B^s(h)'}{h}z_t$$  (15)

Since we have the 5-1 yield spread, we can use it to obtain the following moment restriction:

$$e'_{y1} + e'_{yspr} = \frac{1}{5} e'_{y1} (I - \tilde{\Psi}^5)(I - \tilde{\Psi})^{-1}$$  (16)

$$y^s_{0,1} + yspr_0 = -\frac{1}{5} A^s$$  (17)

where $\tilde{\Psi} = \Psi - \Sigma^{\frac{1}{2}}\Lambda_1$. Then we use equation (1) to fit yields of 2, 10, 20, 30 years from the data. For that we need to calculate the $A^s, B^s$.

$$A^s_{r+1} = -y^s_{0,1} + A^s_r + \frac{1}{2}(B^s_r)'\Sigma(B^s_r) - (B^s_r)'\Sigma^{\frac{1}{2}}\Lambda_0$$  (18)

$$ (B^s_{r+1})' = (B^s_r)'\Psi - e'_{y1} - (B^s_r)'\Sigma^{\frac{1}{2}}\Lambda_1$$  (19)

Thus, we have 12 restrictions from equation (2), 1 from equation (3), and $4 \times T$ restrictions from equations (1),(4),(5). We overweight the 30 year yield.

Real Bond Pricing

The real bond yields are also affine in the state vector:

$$y_t(h) = -\frac{A(h)}{h} - \frac{B(h)'}{h}z_t$$  (20)

We match data for real bond yields for 5, 7, 10, 20 and 30 years using the above expression and using:

$$A(h + 1) = -y_0(1) + A(h) + \frac{1}{2}(B(h))'\Sigma(B(h)) - (B(h))'\Sigma^{\frac{1}{2}}(\Lambda_0 - \Sigma^{\frac{1}{2}} e_{\pi})$$  (21)
\[(B(h + 1))' = -(e_{y1})' + (e_\pi + B(h))'(\Psi - \Sigma^{\frac{1}{2}}\Lambda_1) \]  

(22)

where the real yield is given by:

\[y_0(1) = y_0(1) - y_0 - \frac{1}{2}(e_\pi)'\Sigma e_\pi + (e_\pi)'\Sigma^{\frac{1}{2}}\Lambda_0\]  

(23)

So we have another set of moment conditions, \(T_2 \times 5\). We overweight the 30 year yield.

**Equity Pricing**

The log price-dividend ratios on dividend strips are affine in the state vector:

\[pd_t^n(h) = A^n(h) + (B^n(h))'z_t\]  

(24)

We take the first 3600 dividend strip horizons to match each date’s ratio with the data:

\[\exp(\tilde{pd} + (e_{pd})'z_t) = \sum_{h=0}^{\infty} \exp(A^n(h) + (B^n(h))'z_t)\]  

(25)

where \(\tilde{pd}\) is the mean log price dividend ratio in the data, and the \(A^n(h), B^n(h)\) are:

\[A^n(h + 1) = A^n(h) + \mu^n - y_0(1) + \frac{1}{2}(e_{dgr} + B^n(h))'(e_{dgr} + B^n(h))\]  

\[- (e_{dgr} + B^n(h))'\Sigma^{\frac{1}{2}}(\Lambda_0 - (\Sigma^{\frac{1}{2}})'e_\pi)\]  

(26)

\[B^n'(h + 1) = (e_{dgr} + e_\pi + B^n(h))'\Psi - e'_{y1} - (e_{dgr} + e_\pi + B^n(h))'\Sigma^{\frac{1}{2}}\Lambda_1\]  

(27)

where \(\mu^n\) is the mean dividend growth rate in the data. Finally, we match the equity risk premium using the restriction:

\[(e_{dgr} + \kappa^m_1 e_{pd} + e_\pi)'\Psi - e_{y1}' - (e_{dgr} + \kappa^m_1 e_{pd})'\Sigma^{\frac{1}{2}}\Lambda_1 + e_\pi'\Sigma^{\frac{1}{2}}\Lambda_1\]  

(28)

where \(\kappa^m_1 = \frac{\text{exp}(pd)}{\text{exp}(pd)+1}\). So there are 12 restrictions from equation (14), and \(T \times 1\) restrictions from (11), (12), (13).

The unconditional risk premium can be determined in the following way:

\[r_0^m + \pi_0 - y_{0,1}^S + \frac{1}{2}e_{dgr}'\Sigma e_{dgr} + \frac{1}{2}(e_{dism} + \kappa^m_1 e_{pd})'(e_{dism} + \kappa^m_1 e_{pd}) + e_\pi'(e_{dism} + \kappa^m_1 e_{pd}) = (e_{dism} + \kappa^m_1 e_{pd} + e_\pi)\Sigma^{\frac{1}{2}}\Lambda_0\]  

where \(r_0^m\) is the unconditional mean log real stock return in the data.
B.7.2 Regularization Conditions

Sharpe Ratio

We know that the SDF is exponentially affine and has the following form:

\[ m_{t+1}^S = -y_t^S(1) - \frac{1}{2} \Lambda_t' \Lambda_t - \Lambda_t' \xi_{t+1} \]

It is easy to see that:

\[ E_t m_{t+1}^S = -y_t^S(1) - \frac{1}{2} \Lambda_t' \Lambda_t \]

\[ \text{Var}_t(m_{t+1}^S) = \Lambda_t' \Lambda_t \]

It can be shown that the log price of a n-period bond is given by

\[ p_{nt} = p_{1t} + E_t p_{n-1,t+1} + \frac{1}{2} \text{Var}_t p_{n-1,t+1} + \text{cov}_t(m_{t+1}^S, p_{n-1,t+1}) \]

The Sharpe ratio is defined as the ratio of the expected excess return over the square root of the variance of the excess return. The expected excess return is

\[ E_t p_{n-1,t+1} - p_{nt} + p_{1t} + \frac{1}{2} \text{Var}_t(p_{n-1,t+1}) = -\text{cov}_t(m_{t+1}^S, p_{n-1,t+1}) \]

Therefore, the Sharpe ratio is given by

\[ \theta_t = \frac{-\text{cov}_t(m_{t+1}^S, p_{n-1,t+1})}{\sqrt{\text{Var}_t(p_{n-1,t+1})}} = -\text{cor}_t(m_{t+1}^S, p_{n-1,t+1}) \sqrt{\text{Var}_t(m_{t+1}^S)} \]

The constraint on the maximum Sharpe ratio is (when the correlation term is -1)

\[ \sqrt{\text{Var}_t(m_{t+1}^S)} = \sqrt{\Lambda_t' \Lambda_t} < 1.5 \]

Nominal and Real Yields

We impose that:

\[ y_t^S(h) - y_t(h) \geq (e_x)' \Sigma_\delta \Lambda_0 \]

where we use equations (1) and (6) for the LHS. This restriction is to be tested at maturities of 100, 500, 1000, 2000, 3000, 4000 years.

The following three restrictions are also imposed for the 6 long-term maturities:

1. Nominal yields exceed nominal GDP growth rate. Real yields exceed real GDP
growth rate.

\[ y_t^S(h) \geq .0623 \]

\[ y_t(h) \geq .0304 \]

2. The difference between nominal and real yields must exceed the long run inflation.

\[ y_t^S(h) - y_t(h) \geq .0318 \]

**Bond Return Volatilities**

The bond return is given by:

\[ r_{t+1}^{hs}(h) = \log(P_{t+1}^S(h)) - \log(P_t^S(h + 1)) \]

\[ = A^S(h) - A^S(h + 1) + B^S(h)z_{t+1} - B^S(h + 1)z_t \]

Therefore, the bond return volatility:

\[ Var(r_{t+1}^{hs}(h)) \]

We impose the following condition on bond return volatilities for maturities of 100, 500, 1000, 2000, 3000, 4000 years:

\[ Var(r_{t+1}^{hs}(h)) \geq .2 \]

This can be shown to be equal to

\[ Var(r_{t+1}^{hs}(h)) = B^S(h)'\Sigma B^S(h) \geq .2 \]

**Eigenvalue**

The maximum eigenvalue of the risk neutral companion matrix \( \Psi - \Sigma^{1/2} \Lambda_1 < 1 \).

**B.7.3 SDF Parameter Estimates**

\[
\Lambda_0 = \begin{bmatrix}
0.00 & 3.65 & 6.30 & -10.65 & 0.00 & 3.12 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 33.00
\end{bmatrix}^t
\]
Let \( A_t \) denote the nominal revenue of a local government at date \( t \). We project log growth rates on the state vector

\[
\log \frac{R_{t+1}}{R_t} = R_0 + R^*_t z_t + su_{t+1}
\]  

(29)

Let \( P_t(h) \) denote the price at \( t \) of a strip that pays \( R_{t+h} \) at \( t+h \). Assuming the estimated stochastic discount also prices this assets

\[
\frac{P_t^R(h + 1)}{R_t} = E_t \left[ M_{t+1} P_{t+1}^R(h) \right] 
\]  

(30)

\[
\Rightarrow \frac{P_t^R(h + 1)}{R_t} = E_t \left[ M_{t+1} P_{t+1}^R(h) \frac{R_{t+1}}{R_t} \right] 
\]  

(31)

Conjecture that the log price to dividend ratio of the strip is linear in the aggregate state variables

\[
\log \frac{P_t^R(h + 1)}{R_t} = A^R(h) + B^R(h)' z_t 
\]  

(32)

This yields

\[
\frac{P_t^R(h + 1)}{R_t} = E_t \left[ M_{t+1} P_{t+1}^R(h) \frac{R_{t+1}}{R_t} \right] 
\]  

(33)

\[
= E_t \left[ \exp \left( -y_t (1 - A^R_t - A^R_{t+1}) \right) \right] 
\]  

(34)

\[
= E_t \left[ \exp \left( -y_t (1 - A^R_t - A^R_{t+1} + A^R(h) + B^R(h)' \Psi_{z_t + \Sigma^R \delta_{t+1}} + R_0 + R^*_t \Psi_{z_t} + \Sigma^R \delta_{t+1} + su_{t+1}) \right) \right] 
\]  

(35)

\[
= \exp \left( -y_t (1 - A^R_t + A^R(h) + B^R(h)' \Psi_{z_t + R_0 + R^*_t \Psi_{z_t}}) \right) E_t \left[ \exp \left( (\frac{1}{2} \Lambda^R_t + \frac{1}{2} (B^R(h) + R^*_t) \Sigma^R (B^R(h) + R^*_t) + B^R(h)' \Sigma R^*_t - \Lambda^R_t \frac{1}{2} (B^R(h) + R^*_t)) \right) \right] 
\]  

(36)

\[
\frac{P_t^R(h + 1)}{R_t} = \frac{1}{2} \Lambda^R_t + \frac{1}{2} (B^R(h) + R^*_t) \Sigma^R (B^R(h) + R^*_t) + B^R(h)' \Sigma R^*_t - \Lambda^R_t \frac{1}{2} (B^R(h) + R^*_t) + \frac{1}{2} \Sigma^R
\]  

(37)

B.7.4 Pricing Local Government Claims
where the last equality confirms our conjecture that the log price to dividend ratio is affine in the state vector, since $\Lambda_t$ is affine in $z_t$. As usual within the class of exponentially affine models, it requires the coefficients $A^R(h)$ and $B^R(h)$ to satisfy the recursions

\begin{align}
A^R(h + 1) &= -y_0 + A^R(h) + R_0 + \frac{1}{2}(B^R(h) + R_\beta)'\Sigma(B^R(h) + R_\beta) + B^R(h)'\Sigma R_\beta - \Lambda_0'\Sigma^\frac{1}{2} (B^R(h) + R_\beta) + \frac{1}{2}s^2 \\
B^R(h + 1)' &= -e_{y_1} + (B^R(h) + R_\beta)'\Psi - (B^R(h) + R_\beta)'\Sigma^\frac{1}{2}\Lambda_1
\end{align}

with the boundary conditions

\begin{align}
A(0) = 0, B(0)' = 0
\end{align}

The price to dividend ratio of the full revenue stream therefore is

\begin{align}
\frac{P_t^R}{R_t} = \sum_{h=1}^{\infty} \frac{P_t(h)}{R_t}
\end{align}
Data Appendix

DA.7.1 Annual Survey of State and Local Government Finances

We perform extensive comparisons of the time series as constructed from the Annual Survey of State and Local Government Finances (ASSLGF) and BEA NIPA. For that we aggregate the cross-section of state and local governments and compare the resulting time series against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Below the most relevant items are shown. The full set of comparisons can be obtained from the authors upon request.
Notes: Panels plot the time series of tax revenues from the Census Bureau Annual Survey of State and Local Government Finances against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Years ending on "2" and "7" are full census years. In the intermediate period only a subset of observations are observed; remaining missing values are interpolated according to the Census Bureau’s interpolation method. Taxes on production and imports include property tax, sales tax, excise tax, and other taxes on production and imports. Personal taxes subsume personal income tax and personal other taxes.
Figure DA.2: NIPA and ASSLGF Consumption Expenditures

Notes: The figure plots the time series of consumption expenditures from the Census Bureau Annual Survey of State and Local Government Finances against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Years ending on "2" and "7" are full census years. In the intermediate period only a subset of observations are observed; remaining missing values are interpolated according to the Census Bureau’s interpolation method. Consumption expenditures include current expenditures on fire protection, parks and recreation, natural resources, corrections, hospitals, health expenditures, other current expenditures, primary and secondary education, higher education, education n.e.c., central staff expenditures, judicial, libraries, financial administration, solid waste, general building, police, and protective inspection.

Figure DA.3: NIPA and ASSLGF Gross Investment

Notes: The figure plots the time series of consumption expenditures from the Census Bureau Annual Survey of State and Local Government Finances against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Years ending on "2" and "7" are full census years. In the intermediate period only a subset of observations are observed; remaining missing values are interpolated according to the Census Bureau’s interpolation method. Gross investment includes capital investments for port facilities, water utilities, highways, air transport, and capital expenditures n.e.c., natural resources, parks and recreation, education, protective and inspection, solid waste, corrections, libraries, general buildings, parking facilities, liquor stores, transit utilities, sewage, electric utilities, fire protection, central staff, health infrastructure, policy, housing, judicial, financial administration, and gas utilities.
DA.7.2 Municipal Shapefiles

Shapefiles for municipalities are not readily available. We construct shape files at the municipal level across the United States by combining information from the Census of Government and shape files from the Census Bureau for places and county subdivisions.

We proceed as follows: First, we select all city and town governments from the Census of Government, that is, units with unit indicator 2 and 3. These city and town governments have a self-governing structure which allow the execution of governmental and administrative functions. As cities were founded and developed throughout the history of the United States, the Census Bureau added additional statistical units by necessity.\(^{20}\) As a result, there is no uniform statistical unit that reflects all city and town governments. Nevertheless, some patterns have emerged.\(^{21}\)

In the northeast and midwest incorporated townships often correspond to county subdivisions.\(^{22}\) In the remainder of the United States, the local governments typically correspond to Census places—i.e., urban agglomerations with a self-governing structure.

<table>
<thead>
<tr>
<th>Geographical Unit</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Census Place</td>
<td>19,393</td>
</tr>
<tr>
<td>Census County Subdivision</td>
<td>16,113</td>
</tr>
<tr>
<td>Total</td>
<td>35,506</td>
</tr>
</tbody>
</table>

Table DA.1: Summary Statistics Geographies

\(^{20}\)The Census Bureau defines two major statistical/geographical areas at the sub-county level: (i) minor civil divisions (MCDs) and (ii) census county divisions (CCDs). While minor civil divisions have legal boundaries and names, as well as, governmental functions or administrative purposes specified by state law, census county divisions are county division mainly for statistical purposes. Many states in the southern and western parts of the United States had few sub-county governmental units; as a result, census county division were introduced starting in the 1950s.

\(^{21}\)A detailed description and chronology is provided in [https://www2.census.gov/geo/pdfs/reference/GARM/Ch8GARM.pdf](https://www2.census.gov/geo/pdfs/reference/GARM/Ch8GARM.pdf)

\(^{22}\)A detailed correspondence is tabulated in [https://www2.census.gov/geo/pdfs/reference/GARM/Ch8GARM.pdf](https://www2.census.gov/geo/pdfs/reference/GARM/Ch8GARM.pdf), Table 8-2. In the northeast and midwest, local governments simultaneously correspond to a Census place and a Census subdivision. In those cases, we found that the geographical delineation coincides
Supplementary Appendix

SA.7.1 Geographic Distribution of National Sample

Figure SA.1: Nationwide Sample

Notes: The sample contains all local governments for which information on the unrestricted net position is available in 2007 and 2018. Shape files for local governments are self-constructed as described in the Data Appendix DA.7.2. Data is obtained from the Census of Governments, the Census Bureau and Moody’s Investor Services.

SA.7.2 Census Certainty Sample
Figure SA.2: Certainty Sample - GZ Spread - Unrestricted Net Position

Notes: Panel (a) plots the relationship between the GZ Spread (Gilchrist and Zakrajšek, 2012), a duration matched yield spread, and the unrestricted net position over operating revenues for issuances with maturity of over one year at issuance and tax-exempted status in the primary market. All spreads are tax-rate adjusted. State specific marginal income taxes are obtained from Babina, Jotikasthira, Lundblad, and Ramadorai (2021). Number of observations are restricted to the sample that had at least one primary issuance in the respective fiscal year. Panel (b) plots the relationship between the GZ Spread and the unrestricted net position over operating revenues in the secondary market. Number of observations are restricted to the sample that had at least one transaction in the secondary market in the respective fiscal year. Data sources are detailed in Section 2. All plots are binscatters with 30 quantiles.
Notes: Panel (a) plots the relationship between the GZ Spread (Gilchrist and Zakrajšek, 2012), a duration matched yield spread, and the total liability (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over full value. for issuances with maturity of over one year at issuance and tax-exempted status in the primary market. All spreads are tax-rate adjusted. State specific marginal income taxes are obtained from Babina, Jotikasthira, Lundblad, and Ramadorai (2021). Number of observations are restricted to the sample that had at least one primary issuance in the respective fiscal year. Panel (b) plots the relationship between the GZ Spread and total liability (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over full value in the secondary market. Number of observations are restricted to the sample that had at least one transaction in the secondary market in the respective fiscal year. Data sources are detailed in Section 2. All plots are binscatters with 30 quantiles.